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Existence and Uniqueness of Periodic Solutions for a Prescribed Mean Curvature p- Laplacian Equation with a Deviating Argument

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Abstract: This paper is concerned with the prescribed mean curvature p- Laplacian equation with a deviating argument. By employing Mawhin's coincidence degree theorem and the analysis techniques, some new results on the existence and uniqueness of periodic solutions are obtained. A numerical example demonstrates the validity of the method and the numerical solution diagram is drawn out by MATLAB.

Key words: periodic solution; p- laplacian equation; continuation theorem; prescribed mean curvature

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Introduction

In the past few decades, prescribed mean curvature equations and its modified forms which derived from differential geometry and physics have been drawing considerable attention (see[1－6]). Then, more and more scholars to study the periodic solutions for prescribed mean curvature equation and its modified forms (see[7－10]). For example, Feng in[7] studied the periodic solutions for nonlinear prescribed mean curvature Liénard equations with deviating argument as follows:

$$\left(\frac{x'(t)}{\sqrt{1+(x'(t))^2}}\right)' + f(x(t))x'(t) + g(t, x(t-\tau(t))) = e(t),$$

where $\tau, e \in (R, R)$ are T- periodic, and $g \in C(R \times R, R)$ are T- periodic in the fist argument, $T > 0$ is a constant. Then, Li in[8] discussed a delay prescribed mean curvature Rayleigh equation of the form

$$\left(\frac{x'(t)}{\sqrt{1+(x'(t))^2}}\right)' + f(t, x'(t)) + g(t, x(t-\tau(t))) = e(t),$$

where $\tau, e \in (R, R)$ are T- periodic, and $f, g \in C(R \times R, R)$ are T- periodic in the fist argument, $T > 0$ is a constant.

Recently, by using Mawhin's continuation theorem, Li in[9] studied the existence of periodic solutions for a prescribed mean curvature Liénard p- Laplacian equation with two delays as follows:

$$\left(\varphi_p\left(\frac{x'(t)}{\sqrt{1+(x'(t))^2}}\right)\right)' + f(x(t))x'(t) + g(x(t-\tau(t))) + h(x(t-\gamma(t))) = e(t).$$

Meanwhile, wang in[10] studied the following prescribed mean curvature Rayleigh equation:

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$$\begin{cases}[\varphi_p(\frac{x'(t)}{\sqrt{1+(x'(t))^2}})]'+f(t,x'(t))+g(t,x(t-\tau(t)))=e(t),t\in[0,\omega],\\ x(0)=x(\omega),x'(0)=x'(\omega),\end{cases}\quad(1)$$

under the assumptions:

$$f(t,x)\geqslant a|x|^r,\forall(t,x)\in R^2,$$
$$g(t,x)-e(t)\geqslant -m_1|x|-m_2,\ \forall t\in R,x\geqslant d,$$

where $a,r\geqslant 1$, m_1 and m_2 are positive constants. Through the transformation, (1) is equivalent to the system

$$\begin{cases}x'_1(t)=\frac{\varphi_q(x_2(t))}{\sqrt{1-\varphi_q^2(x_2(t))}},\\ x'_2(t)=-f(t,\frac{\varphi_q(x_2(t))}{\sqrt{1-\varphi_q^2(x_2(t))}})-g(t,x(t-\tau(t)))+e(t),\\ x_1(0)=x_1(\omega),x_2(0)=x_2(\omega).\end{cases}\quad(2)$$

By using Mawhin's continuation theorem and given some sufficient conditions, the authors obtained that (2) has at least one periodic solution. However, we found that the function $\varphi_q(x_2(t))$ must satisfy $\max_{t\in[0,T]}|\varphi_q(x_2(t)))|<1$. That is to say the open and bounded set Ω of Mawhin's continuation theorem must satisfy $\Omega\subset\{(x_1,x_2)^T\in X:|x_1|_0<d,|x_2|_0<\rho<1\}$. But in[10], there is no proof the conditions and a similar problem also occurred in[9].

In order to solve this problem, by using coincidence degree theory and some analysis methods, we study the existence of periodic solutions for prescribed mean curvature p- Laplacian equation with a deviating argument as follows:

$$(\varphi_q(\frac{x'(t)}{\sqrt{1+|x'(t)|^2}}))'+\frac{d}{dt}\nabla F(x(t))+G(x(t-\tau(t)))=e(t),\quad(3)$$

where $p\in(1,+\infty),\varphi_p:R^n\to R^n,\varphi_p(x)=(|x_1|^{p-2}x_1,|x_2|^{p-2}x_2,\cdots,|x_n|^{p-2}x_n)$, for $x\neq 0=(0,0,0)$, $F\in C^2(R^n,R),G\in C(R^n,R^n)$, $e\in C(R,R^n),e(t)=e(t+T),\tau\in C(R,R)$ τ is T- period and $T>0$ is given constant. The existence and uniqueness of periodic solutions to (3) is obtained by using Mawhin's continuation theorem. The interest is that the approaches to estimate a priori bounds of periodic solutions are different from the corresponding ones of[9] and[10]. At last, a numerical example demonstrates the validity of the method.

1 Preliminary

Lemma 1[11] Let L be a Fredholm operator of index zero and let N be L- compact on $\bar{\Omega}$. Suppose that the following conditions are satisfied:

(a1) $Lx\neq\lambda Nx,\forall(x,\lambda)\in\partial\Omega\times(0,1)$;

(a2) $QNv\notin ImL,\forall x\in KerL\cap\partial\Omega$;

(a3) $deg\{JQN,\Omega\cap KerL,0\}\neq 0$, where $Q:Z\to Z$ is a projection with $ImL=KerQ$, $J:ImQ\to KerL$, is an isomorphism with $J(\theta)=\theta$, where θ is the zero element of Z.

Then $Lx=Nx$ has at least one solution in $D(L)\cap\bar{\Omega}$.

Lemma 2[12] Let $0\leqslant\alpha\leqslant T$ be constant, $\tau\in C(R,R)$ be T- periodic function, and $\max_{t\in[0,T]}|\tau(t)|\leqslant\alpha$. Then, $\forall u\in C^1(R,R)$ which is T- periodic function, we have

$$\int_0^T|u(t-\tau(t))-u(t)|^2dt\leqslant 2\alpha^2\int_0^T|u'(t)|^2dt.$$

Lemma 3[13] If $u:R\to R$ is continuously differentiable on R, $a>0,\mu>1$ and $p>1$ are constants, then for every $t\in R$, the following inequality holds:

$$|u(t)|\leqslant(2a)^{-\frac{1}{\mu}}(\int_{t-a}^{t+a}|u(s)|^\mu ds)^{\frac{1}{\mu}}+a(2a)^{-\frac{1}{p}}(\int_{t-a}^{t+a}|u'(s)|^p ds)^{\frac{1}{p}}.$$

This lemma is Corollary 2.1 in[13].

In order to use Mawhin's continuation theorem, we should consider the following system:

$$\begin{cases} x'(t) = \dfrac{\varphi_q(y(t))}{\sqrt{1-|\varphi_q(y(t))|^2}} = \phi(y(t)), \\ y'(t) = -\dfrac{d}{dt}\nabla F(x(t)) - G(x(t-\tau(t))) + e(t). \end{cases}$$

Since

$$\frac{d}{dt}\nabla F(x(t)) = \begin{pmatrix} \dfrac{\partial^2 F}{\partial x_1^2} & \dfrac{\partial^2 F}{\partial x_1 \partial x_2} & \cdots & \dfrac{\partial^2 F}{\partial x_1 \partial x_n} \\ \dfrac{\partial^2 F}{\partial x_2 \partial x_1} & \dfrac{\partial^2 F}{\partial x_2^2} & & \dfrac{\partial^2 F}{\partial x_2 \partial x_n} \\ \vdots & & \ddots & \vdots \\ \dfrac{\partial^2 F}{\partial x_n \partial x_1} & \dfrac{\partial^2 F}{\partial x_n \partial x_2} & \cdots & \dfrac{\partial^2 F}{\partial x_n^2} \end{pmatrix} \begin{pmatrix} x'_1 \\ x'_2 \\ \vdots \\ x'_n \end{pmatrix}$$

and define

$$A = \begin{pmatrix} \dfrac{\partial^2 F}{\partial x_1^2} & \dfrac{\partial^2 F}{\partial x_1 \partial x_2} & \cdots & \dfrac{\partial^2 F}{\partial x_1 \partial x_n} \\ \dfrac{\partial^2 F}{\partial x_2 \partial x_1} & \dfrac{\partial^2 F}{\partial x_2^2} & & \dfrac{\partial^2 F}{\partial x_2 \partial x_n} \\ \vdots & & \ddots & \vdots \\ \dfrac{\partial^2 F}{\partial x_n \partial x_1} & \dfrac{\partial^2 F}{\partial x_n \partial x_2} & \cdots & \dfrac{\partial^2 F}{\partial x_n^2} \end{pmatrix},$$

then the above equation can be turned into

$$\begin{cases} x'(t) = \dfrac{\varphi_q(y(t))}{\sqrt{1-|\varphi_q(y(t))|^2}} = \phi(y(t)), \\ y'(t) = -Ax'(t) - G(x(t-\tau(t))) + e(t), \end{cases} \tag{4}$$

where $\varphi_q(s) = |s|^{q-2}s, \dfrac{1}{p} + \dfrac{1}{q} = 1$, $y(t) = \varphi_p(\dfrac{x'(t)}{\sqrt{1+|x'(t)|^2}} = \phi^{-1}(x'(t))$. Obviously, if $(x(t), y(t))^T$ is a solution of (4), then $x(t)$ is a solution of (3).

Throughout this paper, $|.|$ will denote the absolute value and Euclidean norm on R^n. For each $k \in N$, let $X = Y = \{v = (x(t), y(t))^T \in C(R, R^{2n}), v(t) = v(t+T)\}$, where the norm $\| \int v \| = \max\{|x|_0, |y|_0\}$, and $|x|_0 = \max_{t\in[0,T]} |x(t)|$, $|y|_0 = \max_{t\in[0,T]} |y(t)|$. It is obvious that X and Y are Banach spaces. Furthermore, for $\varphi \in C_T$, $\|\varphi\|_r = (\int_0^T |\varphi(t)|^r)^{\frac{1}{r}}, r > 1$.

Now we define the operator

$$L: D(L) \subset X \to Y, Lv = v' = (x'(t), y'(t))^T,$$

where $D(L) = \{v \mid v = (x(t), y(t))^T \in C^1(R, R^{2n}), v(t) = v(t+T)\}$.

Let $Z = \{v \mid v = (x(t), y(t))^T \in C^1(R, R^n \times \Gamma), v(t) = v(t+T)\}$, where $\Gamma = \{x \in R^n, |x| < 1, x(t) = x(t+T)\}$, define a nonlinear operator $N: \bar{\Omega} \to Y$ as follows:

$$Nv = (\frac{\varphi_q(y(t))}{\sqrt{1-|\varphi_q(y(t))|^2}}, -A\phi(y(t)) - G(x(t-\tau(t))) + e(t))^t,$$

where $\Omega \subset (X \cap Z) \subset X$ and Ω is an open and bounded set. Then problem (2.1) can be written as $Lv = Nv$ in $\bar{\Omega}$.

We know

$$KerL = \{v \mid v \in X, v' = (x'(t), y'(t))^T = (0,0)^T\},$$

then $x'(t)=0, y'(t)=0$, obviously $x\in R^n$, $y\in R^n$, thus $KerL=R^{2n}$, and it is also easy to prove that $ImL=\{z\in Y, \int_0^T z(s)ds=0\}$. So, L is a Fredholm operator of index zero.

Let

$$P: X\to KerL, Pv=\frac{1}{T}\int_0^T v(s)ds,$$

$$Q: Y\to ImQ, Qz=\frac{1}{T}\int_0^T z(s)ds.$$

Let $K_p=L|^{-1}_{Kerp\cap D(L)'}$, then it is easy to see that

$$(K_P z)(t)=\int_0^T G(t,s)z(s)ds,$$

where

$$G(t,s)=\begin{cases}\dfrac{s-T}{T}, 0\leqslant t\leqslant s;\\ \dfrac{s}{T},\ s\leqslant t\leqslant T.\end{cases}$$

For all Ω such that $\bar{\Omega}\subset(X\cap Z)\subset X$, we have $K_P(I-Q)N(\bar{\Omega})$ is a relative compact set of X, $QN(\bar{\Omega})$ is a bounded set of Y, so the operator N is L- compact in $\bar{\Omega}$.

2 Main results

Firstly, we give the following assumptions:

[H_1] There exists a constant $m_1>0$ such that $\langle x, G(x)\rangle\leqslant -m_1|x|^2, \forall x\in R^m$, and $G'(x)<0$, $\forall x\in R^n$.

[H_2] There exists a constant $l>0$ such that $|G(x_1)-G(x_2)|\leqslant l|x_1-x_2|, \forall x_i\in R^n, i=1,2$.

[H_3] There exist constants $\gamma>0, m_0>0$ such that $\langle Ax, x\rangle\geqslant\gamma|x|^2$ and $|Ax|\leqslant m_0|x|, \forall x\in R^n$.

[H_4] $\langle G(x_1)-G(x_2), x_1-x_2\rangle<0, \forall x_1, x_2\in R^n, x_1\neq x_2$.

Theorem 1 If the conditions [H_1]-[H_3] hold, and there exists $\gamma>\sqrt{2}\alpha l$ satisfying

$$(2T)^{\frac{1}{2}}\left(\frac{\gamma^2 T|e|_0^2}{m_1(\gamma-\sqrt{2}\alpha l)^2}\right)^{\frac{1}{q}}+\sqrt{T/2}[m_0+\sqrt{2}\alpha l]\frac{\sqrt{T}|e|_0}{\gamma-\sqrt{2}\alpha l}+T\sqrt{1/2}|e|_0<1,$$

then the problem (4) has at least one periodic solution. Moreover, if [H_4] and $p\geqslant 2$ hold, then the problem (4) has a unique periodic solution.

Proof Let $\Omega_1=\{x\in\Omega, Lx=\lambda Nx, \forall\lambda\in(0,1)\}$. If $\forall\lambda\in\Omega_1$, we have

$$\begin{cases}x'(t)=\lambda\dfrac{\varphi_q(y(t))}{\sqrt{1-|\varphi_q(y(t))|^2}}=\lambda\phi(y(t)),\\ y'(t)=-Ax'(t)-\lambda G(x(t-\tau(t)))+\lambda e(t).\end{cases}\tag{5}$$

Multiplying the first equation of (5) by $y'(t)$ and integrating from 0 to T, we have

$$\int_0^T\langle y'(t), x'(t)\rangle dt=\int_0^T\lambda\langle y'(t), \phi(y(t))\rangle dt=\int_0^T\lambda\phi(y(t))dy(t)=0.$$

On the other hand, multiplying the two sides of the second equation of (5) by $x'(t)$ and integrating them over $[0, T]$, we get

$$\int_0^T\langle Ax'(t), x'(t)\rangle dt=\lambda\int_0^T\langle G(x(t-\tau(t))), x'(t)\rangle dt-\lambda\int_0^T\langle e(t), x'(t)\rangle dt.$$

From [H_3], we get

$$\gamma\int_0^T|x'(t)|^2dt$$

$$\leqslant \int_0^T \langle Ax'(t), x'(t)\rangle dt$$
$$= \lambda\int_0^T \langle G(x(t-\tau(t))), x'(t)\rangle dt - \lambda\int_0^T \langle e(t), x'(t)\rangle dt$$
$$= \lambda\int_0^T \langle G(x(t-\tau(t))) - G(x(t)), x'(t)\rangle dt - \lambda\int_0^T \langle e(t), x'(t)\rangle dt$$
$$\leqslant \int_0^T |G(x(t-\tau(t))) - G(x(t))||x'(t)|dt + \int_0^T |e(t)||x'(t)|dt. \tag{6}$$

Combining (6) with [H_2], we have

$$\gamma\int_0^T |x'(x)|^2 dt \leqslant l\int_0^T |x(t-\tau(t)) - x(t)||x'(t)|dt + \int_0^T |e(t)||x'(t)|dt,$$

by using Hölder's inequality and Lemma 1 to the above inequality, we obtain

$$\gamma \|x'\|_2^2 \leqslant l(\int_0^T |x(t-\tau(t)) - x(t)|^2 dt)^{\frac{1}{2}}(\int_0^T |x'(t)|^2 dt)^{\frac{1}{2}}$$
$$+ (\int_0^T |e(t)|^2 dt)^{\frac{1}{2}}(\int_{|}^T x'(t)|^2 dt)^{\frac{1}{2}}$$
$$\leqslant \sqrt{2}\alpha l\|x'\|_2^2 + \|x'\|_2\|e\|_2,$$

which implies that

$$\|x'\|_2 \leqslant \frac{\sqrt{T}|e|_0}{\gamma - \sqrt{2}\alpha l} := d_0. \tag{7}$$

Multiplying the second equation of (5) by $x(t)$ and integrating from 0 to T, we have

$$\int_0^T \langle x(t), y'(t)\rangle dt$$
$$= \lambda[-\int_0^T \langle x(t), A\frac{x'(t)}{\lambda}\rangle dt - \int_0^T \langle x(t), G(x-\tau(t)))\rangle dt + \int_0^T \langle x(t), e(t)\rangle dt],$$

i.e.,

$$\int_0^T \frac{|y(t)|^q}{\sqrt{1-|\varphi_q(x_2(t))|^2}}dt$$
$$= \int_0^T \langle x(t), (G(x(t-\tau(t))) - G(x(t)))\rangle dt + \int_0^T \langle x(t), G(x(t))\rangle dt - \int_0^T \langle x(t), e(t)\rangle dt$$
$$\leqslant \int_0^T |x(t)||G(x(t-\tau(t))) - G(x(t))|dt$$
$$+ \int_0^T \langle x(t), G(x(t))\rangle dt + \int_0^T |x(t)||e(t)|dt. \tag{8}$$

Combining (8) with [H_1] and [H_2], we get

$$\|y\|_q^q + m_1\|x\|_2^2 \leqslant l\int_0^T |x(t)||x(t-\tau(t)) - x(t)|dt + \int_0^T |x(t)||e(t)|dt.$$

By using Hölder's inequality and Lemma 2 to the above inequality, we obtain

$$\|y\|_q^q + m_1\|x\|_2^2 \leqslant \sqrt{2}\alpha l\|x'\|_2\|x\|_2 + \|e\|_2\|x\|_2,$$

which implies that

$$m_1\|x\|_2^2 \leqslant \sqrt{2}\alpha l\|x'\|_2\|x\|_2 + \|e\|_2\|x\|_2, \tag{9}$$

and

$$\|y\|_q^q \leqslant \sqrt{2}\alpha l\|x'\|_2\|x\|_2 + \|e\|_2\|x\|_2. \tag{10}$$

So from (7), (9) and [H_3], we can conclude that

$$\|x\|_2 \leqslant \frac{\gamma\sqrt{T}|e|_0}{m_1(\gamma - \sqrt{2}\alpha l)} := d_1. \tag{11}$$

Thus by using Lemma 3 for $t \in [0, T]$, we get

$$|x(t)| \leqslant (T)^{-\frac{1}{2}}(\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} |x(s)|^2 ds)^{\frac{1}{2}} + T(T)^{-\frac{1}{2}}(\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} |x'(s)|^2 ds)^{\frac{1}{2}}$$

$$= (T)^{-\frac{1}{2}}(\int_{\frac{T}{2}}^{\frac{T}{2}} |x(s)|^2 ds)^{\frac{1}{2}} + T^{\frac{1}{2}}(\int_{\frac{T}{2}}^{\frac{T}{2}} |x'(s)|^2 ds)^{\frac{1}{2}}$$

$$= (T)^{-\frac{1}{2}}(\int_0^T |x(s)|^2 ds)^{\frac{1}{2}} + T^{\frac{1}{2}}(\int_0^T |x'(s)|^2 ds)^{\frac{1}{2}}. \quad (12)$$

From (7), (11) and (12), we obtain

$$|x|_0 = \max_{t\in[0,T]} |x(t)| \leqslant (2T)^{-\frac{1}{2}} d_1 + \sqrt{\frac{T}{2}} d_0 := \rho_0. \quad (13)$$

From (7), (10) and (11), we obtain

$$\| y \|_q \leqslant (\frac{\gamma^2 T |e|_0^2}{m_1(\gamma - \sqrt{2}\alpha l)})^{\frac{1}{q}} := d_2.$$

Multiplying the second equation of (5) by $y'(t)$ and integrating from 0 to T, we have

$$\int_{-kT}^{kT} |y'(t)|^2 dt$$

$$= -\int_0^T \langle Ax'(t), y'(t)\rangle dt - \int_0^T \lambda \langle y'(t), G(x(t-\tau(t)))\rangle dt + \int_0^T \lambda \langle y'(t), e(t)\rangle dt$$

$$= -\int_0^T \langle Ax'(t), y'(t)\rangle dt - \int_0^T \lambda \langle y'(t), (G(x(t-\tau(t))) - G(x(t))\rangle dt$$

$$+ \int_0^T \lambda^2 \langle G'(x(t)), \frac{|y(t)|^q}{\sqrt{1 - |\varphi_q(y(t))|^2}} E\rangle dt + \int_0^T \lambda \langle y'(t) e(t)\rangle dt.$$

From $[H_1]$, $[H_2]$ and $[H_3]$, we know that

$$\int_0^T |y'(t)|^2 dt$$

$$\leqslant \int_0^T m_0 |x'(t)| |y'(t)| dt$$

$$+ l\int_0^T |y'(t)| |x(t-\tau(t)) - x(t)| dt + \int_0^T |y'(t)| |e(t)| dt$$

$$\leqslant m_0 \int_0^T |x'(t)| |y'(t)| dt$$

$$+ l\int_0^T |y'(t)| |x(t-\tau(t)) - x(t)| dt + \int_0^T |y'(t)| |e(t)| dt.$$

By using Holder's inequality, Lemma 2 and (13) to the above inequality, we obtain

$$\| y' \|_2^2 \leqslant m_0 \| x' \|_2 \| y' \|_2 + \sqrt{2}\alpha l \| x' \|_2 \| y' \|_2 + \| e \|_2 \| y' \|_2,$$

from (7), we can conclude that

$$\| y' \|_2 \leqslant (m_0 + \sqrt{2}\alpha l) d_0 + \sqrt{T} |e|_0 := d_3. \quad (15)$$

In a similar way to (13), we get

$$|y|_0 = \max_{t\in[0,T]} |y(t)| \leqslant (2T)^{-\frac{1}{2}} d_2 + \sqrt{\frac{T}{2}} d_3 := \rho_1,$$

where

$$\rho_1 = (2T)^{\frac{1}{2}}(\frac{\gamma^2 T |e|_0^2}{m_1(\gamma - \sqrt{2}\alpha l)^2})^{\frac{1}{q}} + \sqrt{T/2}[m_0 + \sqrt{2}\alpha l] \frac{\sqrt{T} |e|_0}{\gamma - \sqrt{2}\alpha l} + T\sqrt{1/2} |e|_0.$$

Since $\rho_1 < 1$, we have

$$|y|_0 \leqslant \rho_1 < 1. \quad (16)$$

Let $G_\rho = \max_{|x|\leqslant\rho_0} |G(x)|$, from (6), we have

$$|x'(t)|_0 \leqslant \lambda \frac{|\varphi_q(y(t))|}{\sqrt{1 - |\varphi_q(y(t))|^2}} \leqslant \frac{\rho_1^{q-1}}{1 - \rho_1^q} := \rho_2, \quad (17)$$

and

$$|y'(t)|_0 \leqslant m_0 |x'(t)| + |G(x(t-\tau(t)))| + |e(t)|$$

$$\leqslant m_0\rho_2 + G_\rho + |e|_0 := \rho_3. \quad (18)$$

Let $\Omega_1 \subset x$ represent the set of all the T- periodic solutions of (5). If $(x,y)^T \in \Omega_1$, by using (13) and (16), we get

$$|x|_0 \leqslant \rho_0,\ |y|_0 \leqslant \rho_1 < 1.$$

Let $\Omega_2 = \{v = (x,y)^T \in KerL,\ QNv = 0\}$, if $(x,y)^T \in \Omega_2$, then $(x,y)^T = (a_1,a_2)^T \in R^{2n}$ (constant vector), we see that

$$\begin{cases} \int_0^T \frac{\varphi_q(a_2)}{\sqrt{1-|\varphi_q(a_2)|^2}} dt = 0, \\ \int_0^T [-G(a_1) + e(t)] dt = 0, \end{cases}$$

i.e.,

$$\begin{cases} a_2 = 0, \\ \int_0^T -G(a_1) + e(t) dt = 0. \end{cases} \tag{19}$$

Multiplying the second equation of (19) by a_1, we have

$$Tm_1 a_1^2 \leqslant \int_0^T \langle a_1, e(t) \rangle dt \leqslant T|a_1||e|_0, \tag{20}$$

thus

$$|a_1| \leqslant \frac{|e|_0}{\sqrt{T}m_1} := \beta.$$

Now, if we set $\Omega = \{v = (x,y)T \in X_k,\ |x|_0 < \rho_0 + \beta,\ |y|_0 < \rho^* < 1\}$, where $\rho^* = \frac{\rho_1 + 1}{2} < 1$, then $\Omega \supset \Omega_1 \supset \Omega_2$. So, condition (a_1) and condition (a_2) of Lemma 1 are satisfied. It remains to verify condition (a_3) of Lemma 1. In order to do this, let

$$H(v,\mu):(\Omega \cap KerL) \times [0,1] \to R^n : H(v,\mu) = \mu(x,y)^T + (1-\mu)JQN(v),$$

where $J: ImQ \to KerL$ is a linear isomorphism, $J(x,y) = (y,x)^T$. From assumption $[H_1]$ and (20), we have

$$v^T H(v,\mu) = (x^2 + y^2) + \frac{1-\mu}{T}\int_0^T [\langle -G(x(t)) + e(t), x(t) \rangle + \frac{|y|^2}{\sqrt{1-\varphi_q(y)}}] dt > 0,$$

$$\forall (v,\mu) \in \partial\Omega \cap KerL \times [0,1].$$

Hence, $v^T H(v,\mu) \neq 0$ for $(v,\mu) \in \partial\Omega \cap KerL \times [0,1]$, which implies

$$\begin{aligned} deg\{JQN, \Omega \cap KerL, 0\} &= deg\{H(v,0), \Omega \cap KerL, 0\} \\ &= deg\{H(v,1), \Omega \cap KerL, 0\} \neq 0. \end{aligned}$$

So condition (a_3) of Lemma 1 is satisfied. Therefore, by using Lemma 1, we see that (5) has one periodic solution. Hence equation (3) has at least one periodic solution in $\bar{\Omega}$.

Now to prove uniqueness, assume that $p \geqslant 2$ and $[H_4]$ holds. Let $x_3(t)$ and $x_4(t)$ be any two solution of (4), and let $y_3(t) = \phi^{-1}(x'_3(t))$ and $y_4(t) = \phi^{-1}(x'_4(t))$. Also, let $u(t) = x_3(t) - x_4(t)$ and $v(t) = y_3(t) - y_4(t)$. We will show that $u(t) \leqslant 0$, $\forall t \in [0,1]$. Suppose there exists a $t_0 \in [0,T)$ such that $u(t_0) = \max_{t\in[0,T]} u(t) = x_3(t_0) - x_4(t_0) > 0$. Then $u'(t_0) = \phi(y_3(t_0)) - \phi(y_4(t_0)) = 0$, which implies that $y_3(t_0) = y_4(t_0)$ and $u''(t_0) \leqslant 0$. But

$$\begin{aligned} u''(t_0) &= \phi'(y_3(t_0)) - \phi'(y_4(t_0)) \\ &= [(\frac{\varphi_q(y_3(t))}{\sqrt{1-|\varphi_q(y_3(t))|^2}}) - (\frac{\varphi_q(y_4(t))}{\sqrt{1-|\varphi_q(y_4(t))|^2}})]_{t=t_0} \\ &= [(\frac{\varphi'_q(y_3(t))}{(1-|\varphi_q(y_3(t))|^2)^{\frac{3}{2}}}) - (\frac{\varphi'_q(y_4(t))}{(1-|\varphi_q(y_4(t))|^2)^{\frac{3}{2}}})]_{t=t_0} \\ &= [\frac{1}{(1-|\varphi_q(y_3(t))|^2)^{\frac{3}{2}}}) - (\varphi'_q(y_3(t)) - \varphi'_q(y_4(t)))]_{t=t_0} \end{aligned}$$

$$= [\frac{1}{(1-|\varphi_q(y_3(t_0))|^2)^{\frac{3}{2}}}) - ((q-1)|y_3(t_0)|^{q-2}y'_3(t_0) - (q-1)|y_4(t_0)|^{q-2}y'_4(t_0))$$

$$= \frac{(q-1)|y_3(t_0)|^{q-2}}{(1-|\varphi_q(y_3(t_0))|^2)^{\frac{3}{2}}}(y'_3(t_0) - y'_4(t_0))$$

$$= \frac{(q-1)|y_3(t_0)|^{q-2}}{(1-|\varphi_q(y_3(t_0))|^2)^{\frac{3}{2}}}v'(t_0)$$

$$= \frac{(q-1)|y_3(t_0)|^{q-2}}{(1-|\varphi_q(y_3(t_0))|^2)^{\frac{3}{2}}}[A(\phi(y_4(t_0)) - A\phi(y_3(t_0))) + G(x_4(t_0-\tau(t_0))) - G(x_3(t_0-\tau(t_0)))]$$

$$= \frac{(q-1)|y_3(t_0)|^{q-2}}{(1-|\varphi_q(y_3(t_0))|^2)^{\frac{3}{2}}}[G(x_4(t_0-\tau(t_0))) - G(x_3(t_0-\tau(t_0)))]$$

$$\geqslant 0,$$

which is a contradiction. Hence $\max_{t\in[0,T]} u(t) \leqslant 0$. Similarly, exchanging the role of x_3 and x_4, we can show that $\max_{t\in[0,T]} u(t) \geqslant 0$. This implies that $u(t) \equiv 0$. Therefore, the problem (4) has at most one solution. The proof of Theorem 1 is now complete.

3 Application

As an application, we consider the following example:

$$(\varphi_3(\frac{x'(t)}{\sqrt{1+|x'(t)|^2}})) + \frac{d}{dt}\nabla F(x(t)) - \begin{pmatrix} x_1(t-\frac{\cos(100t)}{3\pi}) \\ x_2(t-\frac{\cos(100t)}{3\pi}) \end{pmatrix} = \begin{pmatrix} \frac{1}{100}\sin(100t) \\ \frac{3}{100}\sin(100t) \end{pmatrix}. \tag{21}$$

Corresponding to Theorem 1, we have $p=3, F(x)=x^2=x_1^2+x_2^2, G(x)=-x, e(t)=\begin{pmatrix} \frac{1}{100}\sin(100t) \\ \frac{3}{100}\sin(100t) \end{pmatrix}$ and $\tau(t)=\frac{\cos(100t)}{3\pi}$, then $\frac{d}{dt}\nabla F(x(t)) = Ax'(t) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix}$, $q=\frac{3}{2}$, $T=\frac{\pi}{50}$, $l=m_1=\alpha=c=1$, $m_2=5$, $\gamma=2$, $m_0=2$, and

$$(2T)^{\frac{1}{2}}(\frac{\gamma^2 T|e|_0^2}{m_1(\gamma-\sqrt{2}\alpha l)^2})^{\frac{1}{q}} + \sqrt{T/2}[m_0+\sqrt{2}\alpha l]\frac{\sqrt{T}|e|_0}{\gamma-\sqrt{2}\alpha l} + T\sqrt{1/2}|e|_0 \approx 0.2417 < 1.$$

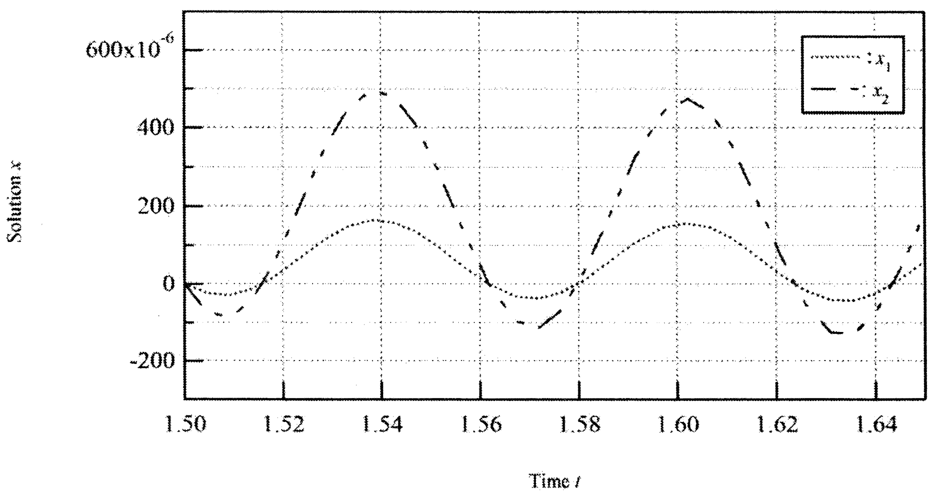

Figure 1. Example with time-varying delay

Hence, by using Theorem 1, we see that (21) has at least $\frac{\pi}{50}$- periodic solution, which can also be illustrated by numerical simulation. By using MATLAB(R2013a) toolkit: , which can be used to solve time-varying delay differential equations, (21) is simulated on tspan = [1.5, 1.65] with Figure 1: Example with time-varying delayhistory = [0,0]. It can be found from Figure 1 that the equation admits one periodic solution with periodicity 0.0628, which is around $\frac{\pi}{50}$. Therefore, the results achieved in this paper are significant.

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一类时滞平均曲率 *p*- Laplacian 方程的周期解存在性与唯一性

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摘　要:这篇文章主要讨论了一类时滞平均曲率 *p*- Laplacian 方程. 通过运用重合度理论和一些分析技巧，得到此类方程周期解存在性与唯一性相关结论. 我们给出相应的数字例子说明其方法以及给出条件的有效性并用 MATLAB 软件画出其数值解图.

关键词:周期解; *p*- Laplacian 方程;重合度理论;平均曲率